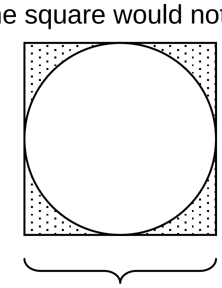
Probability, Statistics, & Data Analysis		
Counting Principle – If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then event *M* followed by the event *N* can occur in $m \cdot n$ ways. • Draw a tree diagram if the number of possibilities doesn't seem clear. • Consider whether or not what happens in the first event effects the second.	1. Joe goes to restaurant for a dinner consisting of soup, a main dish, and a dessert. If the menu has chicken noodle soup, vegetable soup, steak, grilled chicken, lasagna, chocolate cake, and strawberry pie listed as options, how different combinations does Joe have to choose from?	
	2. How many four-letter patterns can be formed using the letters *x, y,* and *z* if letters may be repeated?	
	3. How many four-letter patterns are can be formed by *A, B, C,* and *D* if each letter is used exactly once?	
Permutations – choosing when *order matters* • Use $_nP_r$ on calc (MATH – PRB – 2) • $n =$ total number to choose from • $r =$ number you are choosing • Ex. If $n = 10$ and $r = 3$, type $10\,_nP_r\,3$ and get 720.	4. How many ways could 1st, 2nd, and 3rd place awards be determined in a race with 15 competitors?	
	5. There are 40 people in the senior class. If everyone is in the running, how many ways are there to choose a president, vice president, secretary, and treasurer for the class?	
Combinations – choosing *order does not matter* • Use $_nC_r$ on calc (MATH – PRB – 3)	6. How many ways can 4 committee members be chosen from a group of 30 people?	
	7. How many different groups of 5 could be formed in a class of 20 students?	
Basic Probability – • $\text{probability} = \dfrac{\text{possibilities desired}}{\text{total possibilities}}$ • Always between 0 and 1 • The probability of an event happening plus the probability that it won't happen always equals 1.	8. A bag contains 6 white, 3 blue, and 7 green marbles. If one marble is chosen at random, what is the probability that it is not green?	
	9. Suppose three letters are selected from the word *arrangements.* What is the probability of randomly selecting three consonants?	
	10. A circle is inscribed in a square with a radius of one. Find the probability that a point selected at random from within the square would not be in the circle?  1 in	

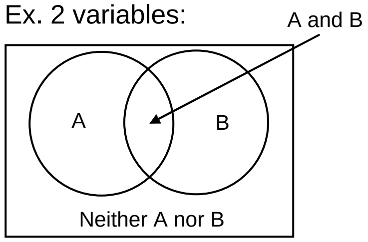
Answers:

1) 12 2) 81 3) 24 4) 2730 5) 2,193,360 6) 27,405 7) 15,504 8) $\frac{9}{16}$ 9) $\frac{14}{55}$ 10) $1 - \frac{\pi}{4}\, in^2$

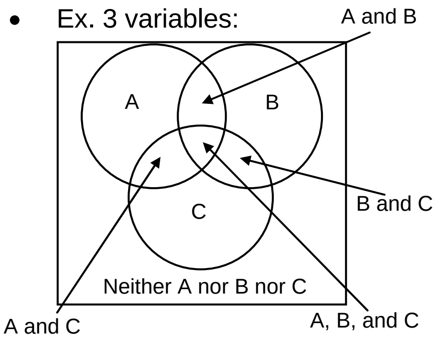
Venn Diagrams

a. Draw a circle for each variable that overlaps each of the other circles. Draw a box around the circles.

b. Ex. 2 variables:



- Ex. 3 variables:



- Plug in any given information and then use addition and subtraction to find the missing values.

1. In a class of 50 students, 18 take Chorus, 26 take Band, and 2 take both Chorus and Band. How many students in the class are not enrolled in either Chorus or Band?	
2. Twenty-four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?	
3. A guidance counselor is planning schedules for 30 students. 16 students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. Five want only Latin, and 8 want only Spanish. How many students want French only?	

Answers:
1) 10 2) 3 3) 7

Numbers: Concepts and Properties

Complex Numbers

- $i = \sqrt{-1}$
- $a + bi$ and $a - bi$ are conjugates
- Your calculator has an i button → use it.
- Complex solutions do not appear on graphs of equations.

Matrices

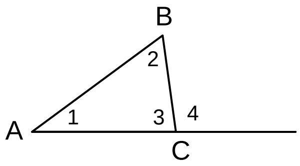
- $\begin{bmatrix} 2 & -1 \\ 3 & 5 \\ 4 & 6 \end{bmatrix}$ is a 3X2 matrix (named by number of rows, then the number of columns)
- Matrices are added by adding the corresponding elements: $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -1 & 5 \end{bmatrix}$. For this to work, they must have the same dimensions.
- Scalar multiplication of a matrix involves multiplying each element by something: $3\begin{bmatrix} 2 & -3 & 4 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 12 \\ -6 & 3 & 9 \end{bmatrix}$

Patterns

- One popular type of problem on standardized tests involves defining some unknown symbol. For example, the question might define an operation $a \infty b = a^2 + 2b$ and ask you to evaluate $4 \infty 3$. To do this, simply follow the pattern defined for you: $4 \infty 3 = 4^2 + 2(3) = 22$.

Properties of Plane Figures + Measurement

Triangles

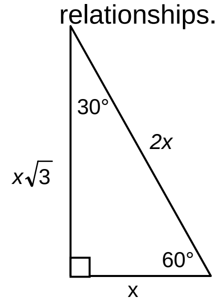


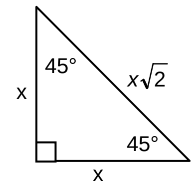
- $m\angle 1 + m\angle 2 = m\angle 4$
- $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$
- Any two sides must add up to more than the third side.
- Similar triangles have $\cong$ angles and sides that are in proportion:

  $\Delta ABC \sim \Delta DEF \rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
- There are four ways to prove any triangles are $\cong$: SAS, SSS, ASA, AAS.
- The largest side of a triangle is always opposite the largest angle.
- Triangles can be classified by sides (scalene, isosceles, equilateral) or by angles (acute, right, obtuse).
- $Area = \frac{1}{2}bh$

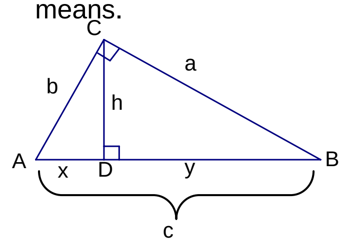
Right Triangles

- The Pythagorean theorem can be used for right triangles: $a^2 + b^2 = c^2$.
- There are 4 ways to prove right triangles are $\cong$: HL, LL (SAS), LA (AAS/ASA), HA (AAS).
- 30-60-90 and 45-45-90 triangles exhibit special relationships.





- Given a right triangle with an altitude, 3 similar triangles are formed, creating 3 geometric means.



$\frac{x}{h} = \frac{h}{y}$

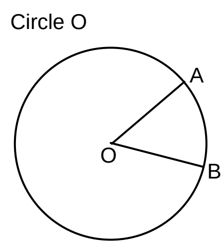
$\frac{y}{a} = \frac{a}{c}$

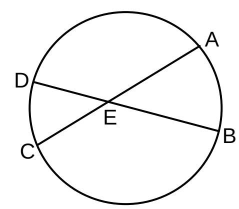
$\frac{x}{b} = \frac{b}{c}$

Circles

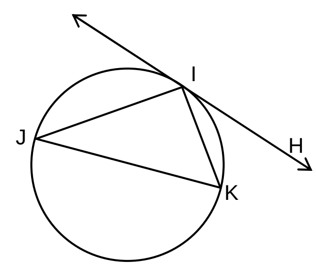
- A circle is named by its center.
- $Area = \pi r^2$; $Circumference = 2\pi r = \pi d$
- All radii are equal in measure: $OA = OB$.
- A central angle has the same measure as its intercepted arc: $m\angle AOB = m\overset{\frown}{AB}$.
- Vertical angles formed by lines intersecting in the circle measure half the sum of the intersected arcs:

  $m\angle DEC = \frac{1}{2}\left(m\overset{\frown}{CD} + m\overset{\frown}{AB}\right)$.
- $DE \cdot EB = CE \cdot EA$





- Inscribed angles and those formed by a tangent and chord intersecting on the circle measure half the intercepted arc: $m\angle IJK = m\angle HIK = \frac{1}{2}m\overset{\frown}{IK}$.

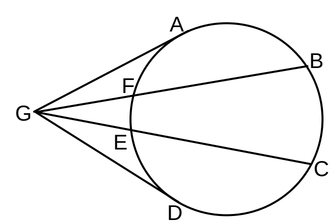


- Angles formed by combinations of secants and tangents intersecting outside the circle measure half the difference of the intercepted arcs:

  $m\angle AGD = \frac{1}{2}\left(m\overset{\frown}{ABD} - m\overset{\frown}{AD}\right)$, $m\angle AGC = \frac{1}{2}\left(m\overset{\frown}{AC} - m\overset{\frown}{AE}\right)$

  $m\angle BGC = \frac{1}{2}\left(m\overset{\frown}{BC} - m\overset{\frown}{EF}\right)$
- When segments are drawn from an exterior point tangent or secant to a segment, the product of the measures of the external segments and the entire segments are equal to each other:

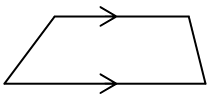
  $GA^2 = GF \cdot GB = GE \cdot GC = GD^2$.



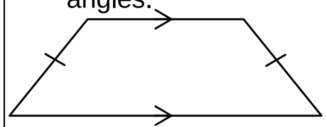
Quadrilaterals: 4-sided figures

Trapezoids

- 1 pair parallel sides
- median connects the midpoints
- $median = \frac{1}{2}(b_1 + b_2)$
- $A = \frac{1}{2}(b_1 + b_2)h$

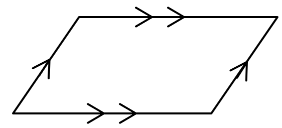


- Isosceles trapezoids have congruent legs and two pairs of congruent base angles.



Parallelograms

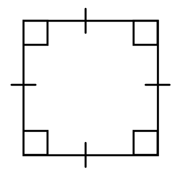
- opposite sides parallel
- opposite sides congruent
- opposite angles congruent
- consecutive angles supplementary
- diagonals bisect each other
- $A = bh$



Rectangles

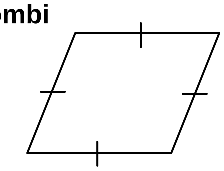
- four right angles
- diagonals congruent

Square



$A = s^2$

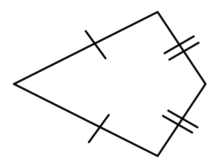
Rhombi



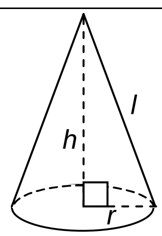
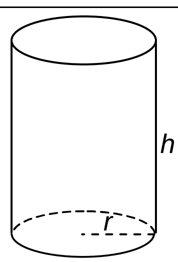
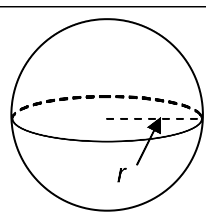
- four congruent sides
- $\perp$ diagonals
- $A = \frac{1}{2}d_1 d_2$

Kites

- two pairs of consecutive sides congruent
- diagonals perpendicular
- one diagonal bisects the other as well as a pair of opposite angles
- $A = \frac{1}{2}d_1 d_2$

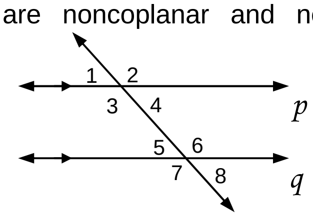


3-Dimensional Figures

	Surface Area	Volume
Cone r = radius l = slant height h = height 	(Area of the Base + Lateral Surface Area) $\pi r^2 + \pi r l$	$\frac{1}{3}\pi r^2 h$
Cylinder r = radius h = height 	(Area of the Base + Lateral Surface Area) $\pi r^2 + 2\pi r h$	$\pi r^2 h$
Sphere r = radius 	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Any **Right Prism** B = area of the base	The sum of the areas of all of the faces	Bh

Other Geometry Tidbits

- 2 points determine a line.
- 3 noncollinear points determine a plane.
- Parallel lines are coplanar and never touch.
- Skew lines are noncoplanar and never touch.

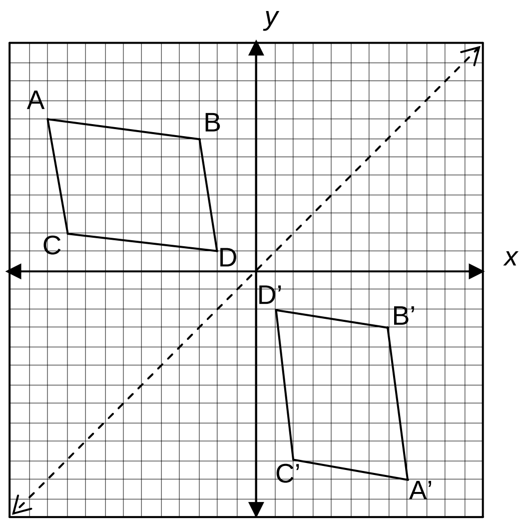


- When parallel lines are cut by a transversal, the following pairs of angles are $\cong$ or supplementary:
  - o Alternate interior ($\angle 4 \cong \angle 5$).
  - o Alternate exterior ($\angle 1 \cong \angle 8$).
  - o Corresponding angles ($\angle 1 \cong \angle 5$).
  - o Consecutive interior angles are supplementary: $m\angle 4 + m\angle 6 = 180$.
- The sum of the interior angle measures of a convex polygon = $180(\#\,sides - 2)$.
- The sum of the exterior angle measures of a convex polygon is 360.
- For a regular polygon, $A = \frac{1}{2}Pa$ where P is the perimeter and a is the apothem (distance from center to a side).
- Concave polygons “cave in”. Convex polygons don’t.

Transformations

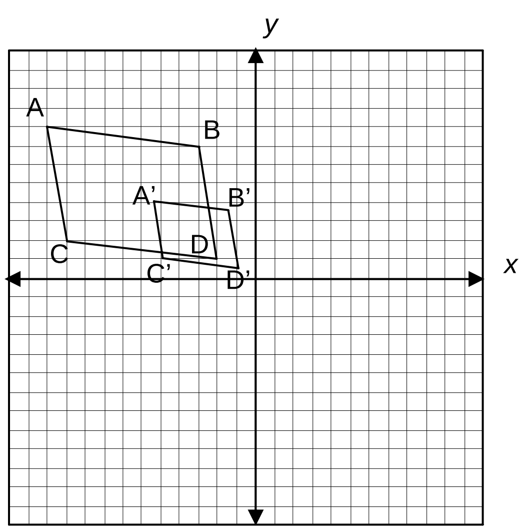
Reflections

- A figure is flipped over a line, producing a mirror image of itself.
- The segment drawn from any point to its reflection must be perpendicular to the line of reflection.
- The preimage and image must be the same distance from the line of reflection
- If the preimage point is A, the image will be called A'.
- In the example below, parallelogram ABCD is reflected over the $y = x$ line.



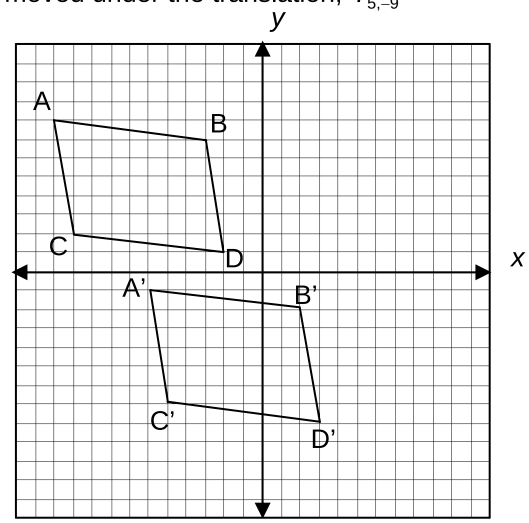
Dilations

- Dilations may be enlargements or reductions, but in each case, the point being dilated is multiplied by a scale factor, which changes its distance from the origin.
- In the example below, parallelogram ABCD is reduced with the dilation $D_{0,\frac{1}{2}}$ (the distance of each point from the origin is divided by two).



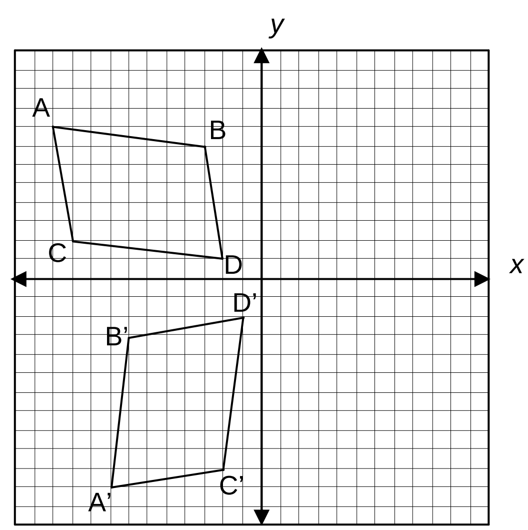
Translations

- A translation *T* maps a point (*x,y*) to (*x*+*a*, *y*+*b*). Essentially, a translation slides a point or figure. Mathematicians would denote this translation with $T_{a,b}$.
- In the example below, parallelogram ABCD is moved under the translation, $T_{5,-9}$



Rotations

- A rotation involves moving a point A around a center point P so that $\overline{AP} = \overline{A'P}$ and $m\angle APA'$ represents the degree of rotation.
- If the rotation in counterclockwise, the degree measure is positive. If it is clockwise, the degree measure is negative
- In the example below, parallelogram is moved with the rotation $\Re_{O,90^\circ}$ (90 degrees counterclockwise about the origin).



Algebraic Expressions

Simplifying and factoring are opposites, so if you are better at one than the other, there is always a chance you can work backwards from the answers.

Simplifying – Put things together

- No parenthesis
- No negative exponents
- No repeated variables that can be combined
- Only identical factors in the numerator and denominator can be cancelled

$a^m a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$\log_b mn = \log_b m + \log_b n$

$\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log_b n^p = p \log_b n$

$(ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^{-n} = \frac{1}{a^n}, \quad \frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ when n is even and a and b are nonnegative

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ all the roots must be defined

$b^{\frac{1}{n}} = \sqrt[n]{b}$ except when $b < 0$ and n is even

$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$ except when $b < 0$ and n is even

$\frac{b-a}{a-b} = -1$

Rational Expressions

- Multiplying and Dividing
  1. Change to multiplication.
  2. Factor everything.
  3. Cancel matching factors from numerator and denominator.
- Adding and Subtracting
  1. Get a common denominator.
  2. Combine like terms in numerator to make one fraction.
  3. Simplify.

Radicals

- Factor the radicand
- Take out factors in groups that match the index.

Factoring – Pull things apart

Greatest Common Factor

- 2 Terms
  - Sum
    - Cubes: $(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$
  - Difference
    - Squares: $(x^2 - y^2) = (x + y)(x - y)$
    - Cubes: $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$
- 3 Terms
  - $ax^2 + bx + c = 0$
  - $\\_\\_\\_ \cdot \\_\\_\\_ = ac$
  - $\\_\\_\\_ + \\_\\_\\_ = b$
  - Split the middle term and group.
- 4 Terms + Grouping

Other tips:

- Factors correspond to zeros on the graph.
- If you know one factor, you can long divide to get others.

Functions

Consider $f(x) = x^2 + 3x - 2$ and $g(x) = x - 3$

- Finding $f(-2)$ involves substituting -2 for x in the function: $f(-2) = (-2)^2 + 3(-2) - 2 = -4$
- Finding $f \circ g(3)$ would mean putting 3 through g, taking the answer and putting that through f.

  $f \circ g(3) = f[g(3)] = f(3-3) = f(0) = 0^2 + 3(0) - 2 = -2$

Equations and Inequalities

- Whatever you do to one side, you must do to the other.
- Any solution to an equation can be checked by plugging the value back into the original equation and making sure both sides are equal.
- A great trick on your calculator is setting the equation equal to zero and seeing where the graph crosses the *x*-axis. This even works for inequalities (look for the part of the x-axis that is shaded and be careful with the boundaries).

Absolute Value Equations

- Isolate the absolute value
- Split the equation into two equations and drop the absolute value sign
- Solve each part
- Check your answer

$2|x-3|+4=12$

$2|x-3|=8$

$|x-3|=4$

$x-3=4$ *or* $x-3=-4$

$x=7$ *or* $x=-1$

Absolute Value Inequalities

- Follow the same steps as you do for an equation, except when you split it consider whether the absolute value is greater than the constant or less than the constant.
    - Less thAND

      $|x|<3$

      $x<3$ *and* $x>-3$

      $-3<x<3$
    - GreatOR

      $|x|>3$

      $x>3$ *or* $x<-3$

Inequalities

- Watch for open ($<$ or $>$) vs. closed ($\le$ or $\ge$) circles.
- Switch the inequality when you divide by a negative

Compound Inequalities

- May be joined by "or" →Any solution to either part must be on the final solution (everything goes)
- May be joined by "and" ($3 \le x \le 5$ can also be written as $3 \le x$ *and* $x \le 5$)→ Only values that are solutions for both parts are in the final solution (overlap).

Quadratics

- Factoring and graphing will work to find the solutions if they are rational.
- Completing the square or the quadratic formula will always work (even if the solutions are irrational or imaginary).
- Put the equation in $ax^2+bx+c=0$ form before applying the quadratic formula

  $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
- For quadratic inequalities, find the solutions to the equation and then see which parts of the x-axis would be shaded if it were graphed as a parabola.

Logarithms

- Use the rules listed under algebraic expressions to set the equation up for a "set =" or a "rewrite"
- "set ="

  $\log_b x=\log_b y \rightarrow x=y$
- "rewrite"

  $\log_b n=p \leftrightarrow b^p=n$
- Use logarithms to solve equations where the variable is an exponent

  $3^x=10$

  $x\log 3=\log 10$

  $x=\dfrac{\log 10}{\log 3}\approx 2.1$
- Remember to eliminate solutions that leave you with the log of a negative number.

Systems of Equations

- Use graphing, substitution, or elimination.
- Make sure the solution(s) work in both equations.

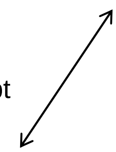
Graphical Representations

- Anything solid or colored in on a graph represents a solution to an equation or inequality**.**
- Dotted lines and open circles represent values that do not actually work in the equation or inequality, but may function as a boundary for the solution set.
- Solutions to systems of equations or inequalities are always where the graphs overlap.
- If you are solving an equation or inequality by graphing, solve for zero, replace 0 with *y*, and look for where the graph crosses the *x*-axis ($y = 0$).
- Functions are undefined for any value that makes the denominator zero.

Lines

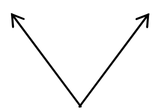
- General/Standard form: $Ax + By = C$ (A, B, and C are all integers)
- Slope-intercept form: $y = mx + b$;

  $slope = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$; b = y-intercept



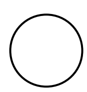
- Point-slope form: $y - y_1 = m(x - x_1)$
- Parallel lines have the same slope.
- Perpendicular lines have slopes that are *negative* reciprocals.
- Horizontal lines have a slope of zero and are written as $y = b$ where b is the y-intercept.
- Vertical lines have no slope and are written $x = c$ where c is the x-intercept.

Absolute Value: $y = a|x - h| + k$ or $x = a|y - k| + h$



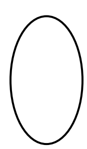
- looks like a "V" in some direction (*a* has the same affect as it does with parabolas)
- (*h, k*) is the vertex.

Circles: $(x - h)^2 + (y - k)^2 = r^2$



- (*h, k*) is the center.
- *r* is the radius.

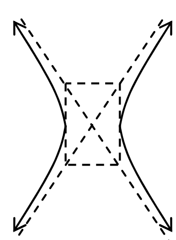
Ellipses: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$



- (*h, k*) is the center.
- Go *a* from the center to edge horizontally.
- Go *b* from the center to the edge vertically.
- Go $c = \sqrt{|a^2 - b^2|}$ to get to the foci on the major (longer) axis.

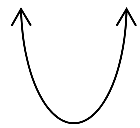
Hyperbolas:

$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$



- (*h, k*) is the center.
- Make a box by figuring out how far to go horizontally and vertically.
- Connect the corners of the box to draw the asymptotes.
- If $x^2 > 0$, it opens side to side.
- If $y^2 > 0$, it opens up and down.
- Draw curves that approach the asymptotes.
- Go $c = \sqrt{a^2 + b^2}$ from the center to the foci on the transverse axis (the one through the vertices).

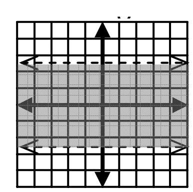
Quadratics/Parabolas



- Vertex form: $y = a(x - h)^2 + k$ (opens up or down and is a function) or $x = a(y - k)^2 + h$ (opens left or right and is not a function)
  - o (*h, k*) is the vertex.
  - o *a* determines
    - ▪ direction of opening
      - If $a > 0$, it opens up or to the right.
      - If $a < 0$, it opens down or to the left.
    - ▪ how wide or narrow the parabola is
  - o The axis of symmetry always goes through the vertex. So it is $x = h$ or $y = k$.
- Standard form: $y = ax^2 + bx + c$
  - o $x = \frac{-b}{2a}$ is the axis of symmetry.

Inequalities

- Watch for dotted/open ($<$ or $>$) vs. solid ($\le$ or $\ge$) conditions.
- Test points and shade toward the ones that work in the inequality.
- $|y| < 2.5$ splits into $y < 2.5$ and $y > 2.5$



Key Equations/Formulas

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- $Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Polynomial Functions

- odd functions - ends go in opposite directions
- even functions - ends go in the same direction
- The degree of the function represents the number of zeros that must be accounted for.
- Every zero corresponds to a factor of the polynomial (if $\frac{3}{2}$ is a zero → $2x - 3$ is a factor).
- Since factors can repeat, zeros can too → one time you will have a repeated zero is when a maximum or minimum is on the x-axis.

Trigonometric Functions

$\sin\vartheta = \dfrac{opp}{hyp}$

$\cos\vartheta = \dfrac{adj}{hyp}$

$\tan\vartheta = \dfrac{\sin\vartheta}{\cos\vartheta} = \dfrac{opp}{adj}$

$\csc\vartheta = \dfrac{1}{\sin\vartheta} = \dfrac{hyp}{opp}$

$\sec\vartheta = \dfrac{1}{\cos\vartheta} = \dfrac{hyp}{adj}$

$\cot\vartheta = \dfrac{1}{\tan\vartheta} = \dfrac{\cos\vartheta}{\sin\vartheta} = \dfrac{adj}{opp}$

- Pythagorean Identities
  - $\sin^2\vartheta + \cos^2\vartheta = 1$
  - $1 + \tan^2\vartheta = \sec^2\vartheta$
  - $\cot^2\vartheta + 1 = \csc^2\vartheta$
- Double-angle Identities
  - $\sin 2\vartheta = 2\sin\vartheta\cos\vartheta$
  - $\cos 2\vartheta = \cos^2\vartheta - \sin^2\vartheta$
    $= 2\cos^2\vartheta - 1$
    $= 1 - 2\sin^2\vartheta$
  - $\tan 2\vartheta = \dfrac{2\tan\vartheta}{1-\tan^2\vartheta}$
- Half-angle Identities
  - $\sin\dfrac{\vartheta}{2} = \pm\sqrt{\dfrac{1-\cos\vartheta}{2}}$
  - $\cos\dfrac{\vartheta}{2} = \pm\sqrt{\dfrac{1+\cos\vartheta}{2}}$
  - $\tan\dfrac{\vartheta}{2} = \pm\sqrt{\dfrac{1-\cos\vartheta}{1+\cos\vartheta}}$
- To solve a problem involving a trig identity, write each function in terms of sine and cosine and simplify. Or, choose a random angle measure to use and replace the variable with it so you can evaluate the expression. Then, evaluate the answer choices and see which one has the same value.
- The amplitude of a function is half the distance from the smallest to the largest value of a function. For $y = a\sin bx$, a is the amplitude.
- The period is how many degrees or radians it takes to make one complete wave.

The Unit Circle

$\sin\vartheta = y$, $\cos\vartheta = x$