ALGEBRA 1 FINAL EXAM STUDY GUIDE! Name:

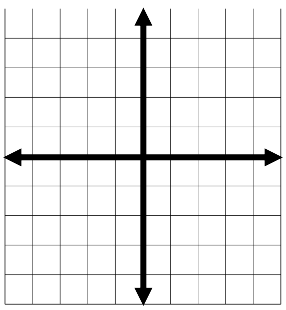
Chapter 6: Systems of Equations and Inequalities

6-1: Solving Systems by Graphing

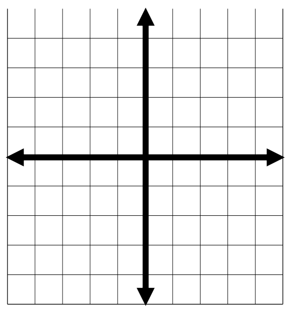
Two or more linear systems together form a **system of linear equations**. Any ordered pair (x, y) in a system that makes all the equations true is a **solution of the system of linear equations**. One method for solving a system is to graph each equation on the same coordinate plane. If the lines intersect, then there is **one solution** to the system. If the lines are parallel, then there is **no solution** to the system. And if the equations produce identical lines, then there are **infinitely many solutions** to the system.

Examples: Solve each system by graphing.

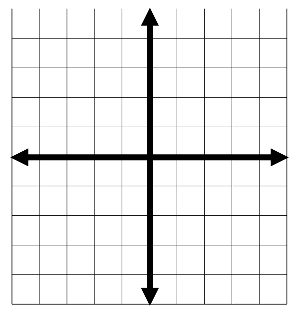
1. $y = -x + 1$
   $y = -2x$



2. $y = \frac{2}{3}x - 1$
   $3y - 2x = -3$



3. $y = \frac{-1}{2}x + 1$
   $2y = -x - 4$



6-2: Solving Systems Using Substitution

Another method for solving systems of equations is the **substitution method**. This method allows you to solve for one variable at a time through series of substitutions.

Steps to the Substitution Method:

1. Isolate a variable in one of the given equations
2. Substitute the expression from Step 1 into the other equation for the variable it equals and solve
3. Substitute the value found in Step 2 into the equation from Step 1 and solve
4. Name the ordered pair solution (x, y)

Examples: Solve each system using substitution.

4. $y = 2x + 11$
   $y = -x + 5$

5. $-4x + y = -3$
   $y = 5x - 1$

6. $2x = -4y - 6$
   $x - 3y = 7$

6-3: Solving Systems Using Elimination

The third method for solving systems of equations is the **elimination method**. This method allows you to eliminate one variable so you can solve for the other variable.

Steps to the Elimination Method:

1. If necessary, multiply one or both of the equations in order to create opposite numbers in front of one of the two variables.
2. Add the two equations together (eliminating the variable with opposite coefficients)
3. Solve the newly formed equation for the remaining variable.
4. Substitute the value from Step 3 into one of the original equations and solve
5. Name the ordered pair solution (x, y)

If both variables are eliminated when the equations are added together, then…

- there is **no solution** if it results in a false statement (like $0 = 4$)
- there are **infinitely many solutions** if it results in a true statement ($0 = 0$)

Examples: Solve each system using elimination.

7. $2x + 9y = 36$
   $2x - y = 16$

8. $x + 8y = 28$
   $-3x + 5y = 3$

9. $5x + 7y = -1$
   $5x + 7y = 6$

Word Problems

A jar contains nickels and dimes. There are 20 coins in the jar and the total value of the coins is $1.40. How many nickels and how many dimes are in the jar?

Julia went to the movies with her friends and bought 3 large popcorns and two small drinks. Her total was $21.00. Jennifer also went to the movies with her friends and bought 2 large popcorns and four small drinks. Her total was $22.00. Find the cost of a large popcorn and the cost of a small drink.

6-4: Solving Special Systems

See section 6.1-6.3

6-5: Linear Inequalities

A **linear inequality** describes a region of the coordinate plane that has a boundary line. The **solutions of a linear inequality** are the ordered pairs that make the inequality true. Graphing a linear inequality in much like graphing a linear equation, along with a couple additional steps.

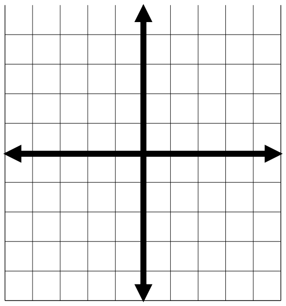
Steps to Graphing Linear Inequalities:

1. Put the inequality in slope-intercept form ($y = mx + b$)
2. Find the slope (m) and the y-intercept (b)

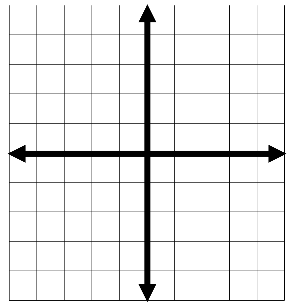
3. Plot b (b means "begin")and then use m (m means "move") to plot additional points (use m to $\frac{rise}{run}$)
4. Connect the points with a dashed or solid line (dashed line for < or >, solid line for the ≤ or ≥).
5. Test a point, not on the line, and shade the side that is "true".

Examples: Graph each linear inequality.

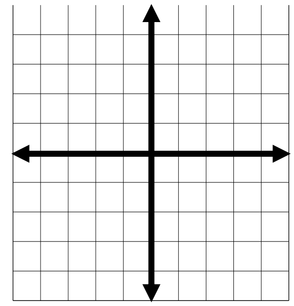
12. $y < 2x - 4$



13. $y \geq -\frac{2}{3}x + 1$



14. $6x + 2y > 8$

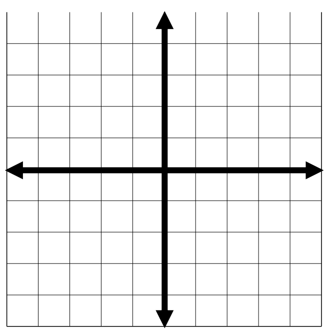


<u>6-6: Systems of Linear Inequalities</u>

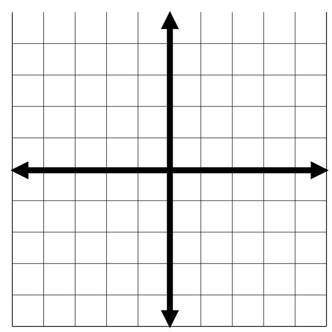
Two or more linear inequalities together form a **system of linear inequalities**. A **solution to a system of linear inequalities** makes each inequality in the system true. Graphically, the solution is the "overlap" of the shaded regions for each linear inequality in the system. By graphing all of the inequalities in the system on the same coordinate plane, you can see where this solution region is located.

Examples: Solve each system of linear inequalities.

15. $\begin{array}{l} y > -3x + 2 \\ y \leq 2x + 2 \end{array}$



16. $\begin{array}{l} x \geq 2 \\ y > -x + 2 \end{array}$



CHAPTER 7: Exponents and Exponential Functions

<u>7-1: Integer Exponents</u>

$a^0 = 1$ - anything to the zero power equals 1

$a^{-n} = \frac{1}{a^n}$ - get rid of all negative exponents ("move and lose" – *move* term with power and *lose* the negative)

Examples to Follow:

A. $(-1.45xy)^0 = 1$

B. $\frac{1}{x^{-3}} = x^3$

C. $3ab^{-2} = \frac{3a}{b^2}$ (move b^2 to bottom)

Evaluate for n = -2 and w = 5.

D. $n^{-3}w^{0} = (-2)^{-3}(5)^{0}$

$$= \frac{1}{(-2)^3}(1)$$

$$= \frac{-1}{8}$$

Practice:

Simplify each expression.

1. $3ab^0$
2. $\frac{6}{t^{-4}}$
3. $\frac{7s}{5t^{-3}}$
4. $\frac{7s^0t^{-5}}{2^{-1}m^2}$

Evaluate for r = -3 and s = 5.

5. s^{-2}
6. $5r^3s^{-1}$
7. r^0s^{-2}
8. $2^{-4}r^3s^{-2}$

7-3: Multiplication Properties of Exponents

$a^m \cdot a^n = a^{m+n}$ (when you multiply with the same base….add the exponents)

$(a^m)^n = a^{mn}$ (when you have a power to a power….multiply the exponents)

Examples to Follow:

E. $x \cdot x^3 \cdot x^{-2} = x^{1+3+(-2)} = x^2$

F. $(5x^5)(3y^6)(3x^2) = (5 \cdot 3 \cdot 3)(x^5 \cdot x^2)(y^6) = 45\ x^7\ y^6$

G. $c^5(c^3)^{-2} = c^5 \cdot c^{-6} = c^{5+(-6)} = c^{-1} = \frac{1}{c}$

H. $(c^2)^3(3c^5)^4 = c^6 \cdot 3^4 \cdot (c^5)^4 = 81 \cdot c^6 \cdot c^{20} = 81 \cdot c^{6+20} = 81c^{26}$

Practice:

9. $c^{-2}c^7$
10. $(4c^4)(ac^3)(3a^5c)$
11. $(x^5y^2)(x^{-6}y)$
12. $5t^{-2} \cdot 2t^{-5}$
13. $(d^3)^5(d^3)^0$
14. $(t^2)^{-2}(t^2)^{-5}$
15. $(2a^2c^4)^{-5}(c^{-1}a^7)^6$
16. $(2y^4)^{-3}$

7-4: Division Properties of Exponents

$$\frac{a^m}{a^n} = a^{m-n}$$ (when you divide with the same base….subtract the exponents)

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$ (distribute the exponent outside to all terms inside)

Examples to Follow:

I. $\frac{a^6}{a^{14}} = a^{6-14} = a^{-8} = \frac{1}{a^8}$

J. $\frac{a^2 b}{a^4 b^3} = \left(\frac{a^2}{a^4}\right)\left(\frac{b}{b^3}\right) = \left(a^{2-4}\right)\left(b^{1-3}\right) = a^{-2}b^{-2} = \frac{1}{a^2 b^2}$

K. $\left(\frac{3}{y^3}\right)^4 = \frac{3^4}{\left(y^3\right)^4} = \frac{81}{y^{12}}$

L. $\left(\frac{4b}{-c}\right)^{-2} = \left(\frac{-c}{4b}\right)^2 = \left(\frac{(-c)^2}{4^2 b^2}\right) = \frac{c^2}{16b^2}$

Practice:

17. $\frac{c^2 d^{-3}}{c^3 d^{-1}}$

18. $\frac{m^{-2}}{m^{-5}}$

19. $\frac{3^2 m^3 t^6}{3^5 m^7 t^{-5}}$

20. $\frac{3^7}{3^{-4}}$

21. $\left(\frac{2x}{y}\right)^5$

22. $\left(\frac{3a}{2b}\right)^4$

23. $\left(-\frac{2}{3}\right)^{-2}$

24. $\left(\frac{4p}{5}\right)^{-3}$

25. $\frac{\left(2a^7\right)\left(3a^2\right)}{6a^3}$

26. $\left(\frac{2k^3}{3k^{-2}}\right)^{-2}$

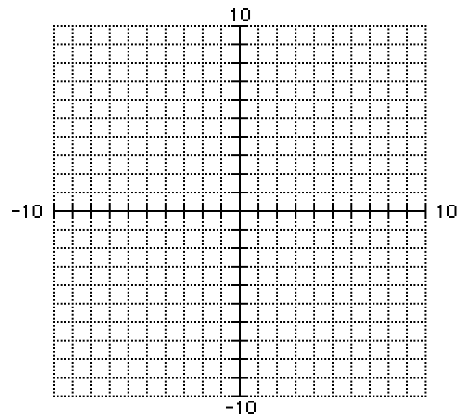
27. $\frac{3^2 \cdot 5^0}{2^{-3}}$

<u>11-2: Exponential Functions</u>

We use a table to graph Exponential Functions.

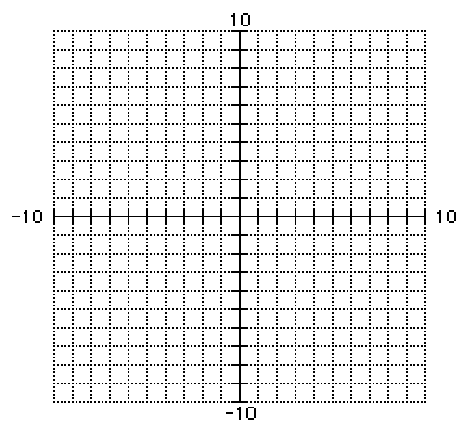
Graph $y = 4 \bullet 2^x$

X	Work	(x,y)



Graph $y = -0.5 \bullet 2^x$

X	Work	(x,y)



<u>8-8: Exponential Growth and Decay</u>

$y = a \cdot b^x$ *a = starting amount, b = growth or decay factor, x = # years*

Evaluate each function rule for the given value.

1. $f(x) = 6^x$ for x = 3

2. $h(w) = 0.5 \cdot 4^w$ for w = 3

3. $y = 9 \cdot \left(\frac{5}{2}\right)^x$ for x = -3

4. Suppose an investment of $10,000 triples in value every 13 years. How much is the investment worth after 52 year? After 65 years?

5. Suppose an investment of $2000 doubles in value every 8 years. How much is the investment worth after 24 years? After 32 years?

6. How much money is in your account after 5 years if a $4000 principal earning 6% is compounded annually?

7. How much money is in your account after 7 years if $12,000 principal earning 4.8% is compounded annually?

8. How much money is in your account after 6 years if $500 principal earning 4% is compounded quarterly?

9. How much money is in your account after 10 years if $20,000 deposit earning 3.5% is compounded quarterly?

7.5 Polynomials:

Degree:____

Degree of monomials:

$-2x^3$ $5x^4y^7$ 8 17x

Polynomial: ____

Degree of a polynomial:

$4x - 18x^3$ $0.5x^2y + 0.25xy - 0.75$ $6x^4 + 9x^2 - x + 3$

Standard Form: ____

Write each polynomial in standard form

$20x - 4x^2 + 2 - x^2$ $-7 + y^3 - 2y^4 + y$ $6x - 7x^5 - 4x^2 + 9$

Special Names

Degree	Name
0	
1	
2	
3	
4	
5	
6 or more	

Terms	Name
1	
2	
3	
4 or more	

<u>7.6 Adding and Subtracting Polynomials</u>:
When you add or subtract terms, you don't do anything to their exponents ($3x + 4x = 7x$ and $3x^2 + 4x^2 = 7x^2$). Just be sure that you are only adding or subtracting LIKE terms.

Simplify.

1. $(4a^2 - 5a) + (3a^2 + 6a - 7)$
2. $(3 + 2a + a^2) - (5 - 8a + a^2)$
3. $(2x^3 - 5x^2 - 1) - (8x^3 + 3 - 8x^2)$

<u>7.7 Multiplying Polynomials</u>: (Distribute and F.O.I.L.)

Rule: When you multiply terms, you _______ their exponents.

Simplify.

4. $8x(4x - 5)$
5. $7xy^2(-2x^2y^3 + 4xy + 1)$
6. $-8s(5s^2 + 3s - 4) + 9s^2(7 + 2s + 4s^2)$
7. $(2x + 3)(5x - 4)$
8. $(2a + 3b)(2a + 3b)$
9. $(4m - 3)(4m + 3)$
10. $(7a + 3b)(7a - 3b)$
11. $(x - 3)^2$
12. $(5x + 4y)^2$

Chapter 8 Factoring Polynomials

Naming

Factor by pulling out the GCF.

1. $10x^2 + 5x - 20$
2. $4x^3 + 8x^2 - 16x$
3. $8x^2y^3 + 20x^2y^2$
4. $v^2 + 4v$

Factor each binomial. Remember to pull out the GCF first if there is one.

5. $x^2 - 36$
6. $25x^2 - 81$
7. $x^4 - 9y^2$
8. $2x^2 - 18$

Factor each trinomial. Remember to pull out the GCF first, if there is one.

9. $d^2 + 10d + 9$
10. $18x^2 + 15x + 3$
11. $a^2 + ab - 56b^2$

Factor each polynomial completely. If it is not factorable, write <u>prime</u>.

14. $r^2 - 49$

15. $6x - 12y$

16. $12d^2 + 4d - 1$

17. $2k^2 - 32h^2$

18. $2m^3 + 6m^2 + 3m + 9$

19. $k^2 + k - 2$

Chapter 9: Quadratic Equations

<u>Section 9.5 –9.9 Solving Quadratic Equations</u>

Recall: Quadratic Equations are simply quadratic functions set equal to zero.

There are 4 methods we've learned for solving quadratic equations

a. Graphing
b. Square Roots
c. Factoring
d. Quadratic Formula

a. **Graphing –**the solutions are where the parabola crosses the x-axis.

b. **Square Roots –** Can only be used when the linear (x) term is missing.
Good for equations in the form $x^2 = c$

Examples: Solve

11. $4x^2 = 100$

12. $2x^2 - 6 = 26$

13. $x^2 + 4 = 0$

c. **Factoring –**To solve by factoring apply the Zero-Product Property
Good to use when the equation is easy to factor

<u>Zero Product Property</u> – If the product of two numbers equals zero, then one of its factors is zero. For example, if (x – 2)(x + 1) = 0 then either (x – 2) = 0 or (x + 4) = 0. This property allows you to solve a quadratic equation

EX: Solve $x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$ **Subtract 3 from each side because quadratic equations always = 0**
$(x - 3)(x + 1) = 0$ **Factor**
$x - 3 = 0$ or $x + 1 = 0$ **Apply the Zero Product Property**
$x = 3$ or $x = -1$ **Solve each factor for x**

Solve by Factoring

14. $x^2 + 7x + 10 = 0$
15. $x^2 - 6x = -8$
16. $x^2 - 3x - 28 = 0$

d. Quadratic Formula - Can be used to solve any quadratic equation
When an equation is in $ax^2 + bx + c = 0$ the solutions can be found by the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the Quadratic Formula to solve each equation. If necessary, round answers to the nearest hundredth.

17. $3x^2 + 7x + 2 = 0$
18. $x^2 + 3x + 2 = 0$
19. $4y^2 = 3 - 5y$

Choose any method and solve

20. $x^2 - 3x - 4 = 0$
21. $-4x^2 + x + 7 = 0$
22. $4x^2 = 25$

Graphing

Graphing Quadratics

General Form: $y = ax^2 + bx + c$

a: controls the direction of opening

+# opens upwards

-# opens downwards

c: y-intercept

To graph:

Step 1. Find the axis of symmetry x= $\frac{-b}{2a}$

Step 2. Find the vertex. Plug the answer from step one into all the x's to find y.

Step 3. Find the y-intercept **C**

Step 4. Find two more points. Pick a point that is close to the vertex. You are actually going to be picking a x-value and plugging it into the equation to find y. That will give you the coordinate. Then reflect the coordinate over the axis of symmetry. Then graph the dots and connect.

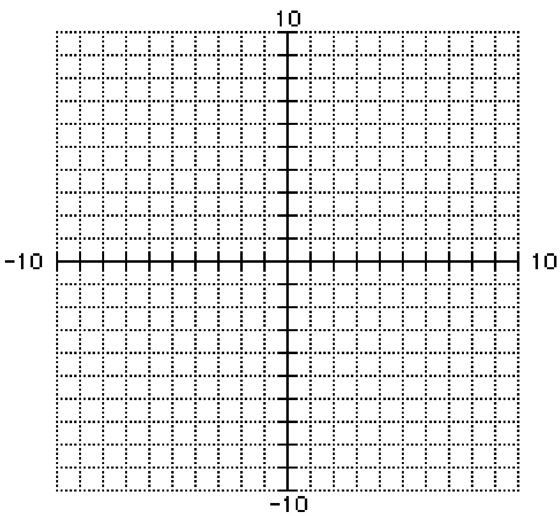
Don't forget inequalities! Don't forget dashed ($<, >$)and solid lines ($\leq, \geq$).

The parabola is upwards : $<$ or $\leq$ shade outside $>$ or $\geq$ shade inside

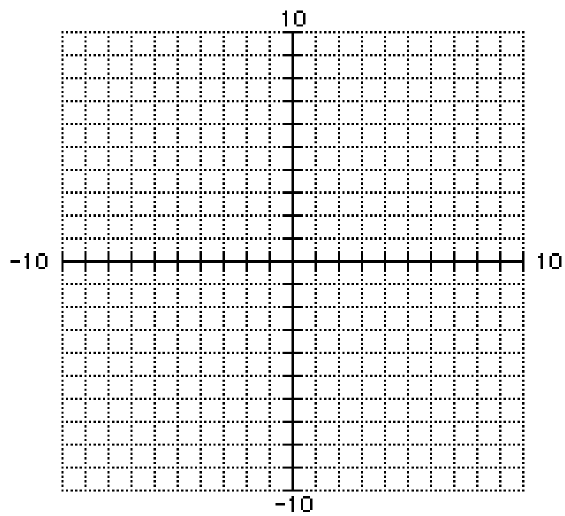
The parabola is downwards : $<$ or $\leq$ shade inside $>$ or $\geq$ shade outside

Graph the inequalities

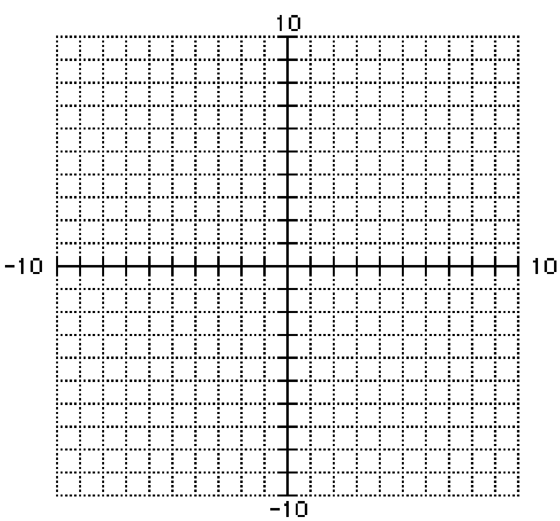
1. $y = x^2$



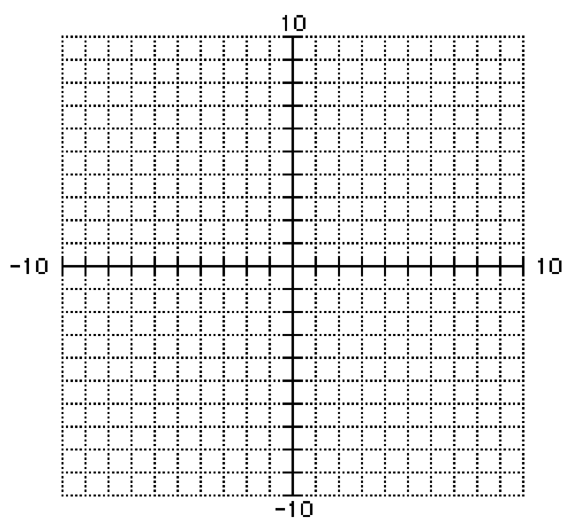
2. $y = -3x^2 - 6$



3. $y < 2x^2 + 6x + 2$



4. $y \geq -4x^2 + 8x + 1$



Chapter 11: Radical Expressions and Equations

11-6-11-9: Simplifying Radicals.

Simplifying a Single Radical. (Do the factor trees for the numbers and look for pairs.)

1. $-2\sqrt{90}$

2. $\sqrt{72x^4}$

3. $-3\sqrt{80x^6y^5}$

Multiplying two Radicals. (Multiply their outsides and insides. Then simplify your answer.)

4. $\sqrt{18}\cdot 2\sqrt{3}$

5. $-\sqrt{50}\cdot\sqrt{5}$

6. $2x\sqrt{5x^2y^3}\cdot 3\sqrt{200x^7}y^3$

Rationalizing the Denominator (Remember, you can not leave a radical sign in the denominator of a fraction.)
Shortcuts you can look for to avoid having to rationalize the denominator.

- If the denominator is a perfect square, just take the square root of the top and the square root of the bottom and simplify.

  Example to follow: $\sqrt{\frac{7}{25}} = \frac{\sqrt{7}}{\sqrt{25}} = \frac{\sqrt{7}}{5}$

- If the fraction under the radical can be simplified, do this first.

  Example to follow: $\sqrt{\frac{18x^5}{2x^2}} = \sqrt{9x^3} = 3x\sqrt{x}$

- If the denominator is not a perfect square and you can't simplify the fraction, then multiply the top and bottom of the fraction by the denominator.

  Example to follow: $\sqrt{\frac{6}{7}} = \frac{\sqrt{6}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{42}}{7}$ (Note: $\sqrt{42}$ cannot be simplified further because its factors are 2·3·7 and there are no pairs.)

Examples: Simplify.

7. $\sqrt{\frac{24}{81x^2}}$

8. $\sqrt{\frac{8x^7}{2x^3}}$

9. $\frac{2}{\sqrt{3}}$

10. $\sqrt{\frac{5}{2x}}$

<u>Operations with Radicals. (Add, Subtract, Distribute and F.O.I.L.)</u>

You can only add or subtract like terms, or like radicals. $2\sqrt{3}+4\sqrt{3}$ can be added together to get $6\sqrt{3}$. However, $2\sqrt{6}+6\sqrt{2}$ cannot be added together because they are not like terms. Sometimes, it looks like you are not able to add two radicals together such as $2\sqrt{12}+5\sqrt{3}$, but you must simplify the radicals first.

$2\sqrt{12}+5\sqrt{3}$ ($2\sqrt{12}$ simplifies to $4\sqrt{3}$)

$= 4\sqrt{3}+5\sqrt{3}$ (now you have like terms, so add.)

$= 9\sqrt{3}$

Examples: Simplify each expression. Leave answers in simplified radical form (not decimals).

11. $-\sqrt{6}+5\sqrt{6}$ 12. $4\sqrt{18}-7\sqrt{2}$ 13. $\sqrt{6}\left(\sqrt{8}-4\right)$

14. $5\sqrt{2}\left(6\sqrt{10}-2\sqrt{6}\right)$ 15. $\left(2\sqrt{3}-5\right)\left(\sqrt{3}+6\right)$ 16. $\left(2\sqrt{5}-4\sqrt{2}\right)^2$

<u>Solve the Radical Equation.</u>

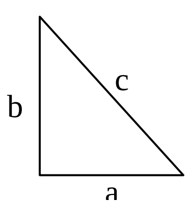
Isolate the radical, then you can square both sides in order to get rid of the square root symbol.

17. $7=\sqrt{v}-1$ 18. $\sqrt{9+x}=4$ 19. $\sqrt{3x+5}=\sqrt{6x-1}$

<u>Extension: Pythagorean Theorem:</u> $\mathbf{a^2 + b^2 = c^2}$

Examples: Given the right triangle, find the missing side. Round your answer to the nearest tenth.

20. a = 2, b = 6 21. b = 8, c = 12 22. a = 5, c = 15



Examples: Determine if the three sides form a right triangle or not.

23. 7, 12, 13 24. 10, 20, 30

<u>Extension: Distance and Mid-Point Formulas</u>

Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Mid-Point Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Examples: Find the **distance** (to the nearest tenth) and the **midpoint** between the two endpoints.

25. (-5, 4) (-7, -7) 26. (-4 , 9) (-2, 5)